

# Casimir-Lifshitz forces and radiative heat transfer between moving bodies.

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Recently Philbin *et al* {New J. Phys. 11 (2009) 033035; arXiv:0904.2148v3 [quant-ph], 2009} have presented a new theory of the van der Waals friction. Contrary to previous theories they claimed that there is no “quantum friction” at zero temperature. We show that this theory is incorrect.

## I. INTRODUCTION

All bodies are surrounded by a fluctuating electromagnetic field due to the thermal and quantum fluctuations of the charge and current density inside the bodies. Outside the bodies this fluctuating electromagnetic field exists partly in the form of propagating electromagnetic waves and partly in the form of evanescent waves. The theory of the fluctuating electromagnetic field was developed by Rytov<sup>1–3</sup>. A great variety of phenomena such as Casimir-Lifshitz forces<sup>4</sup>, near-field radiative heat transfer<sup>5</sup>, and friction forces<sup>6–9</sup> can be described using this theory.

Lifshitz<sup>4</sup> used the Rytov’s theory to formulate a very general theory of the dispersion interaction using statistical physics and macroscopic electrodynamics. The Lifshitz theory provides a common tool to deal with dispersive forces in different fields of science (physics, biology, chemistry) and technology.

The Lifshitz theory is valid for systems at thermal equilibrium. At present there is an interest in the study of systems out of the thermal equilibrium (see<sup>10</sup> and reference therein), in particular in the connection with the possibility of tuning the strength and sign of the interaction<sup>11,12</sup>. Such systems also present a way to explore the role of thermal fluctuations, which usually are masked at thermal equilibrium by the  $T = 0$  K component, which dominates the interaction up to very large distances, where the interaction force is very small. In Ref.<sup>11</sup> the Casimir-Lifshitz force was measured at very large distances, and it was shown that the thermal effects on the Casimir-Lifshitz interaction agree with the theoretical prediction. This measurement was done out of thermal equilibrium, where thermal effects are stronger.

Non-equilibrium thermal effects was also studied by Polder and Van Hove<sup>5</sup>, who calculated the heat-flux between two parallel plates. At present there is an increasing interest in near-field radiative heat transfer<sup>13–18</sup>, in the connection with the development of near-field scanning thermal microscopy<sup>19</sup>. The existing studies are limited mostly to the case when the interacting bodies are at rest. For recent reviews of near-field radiative heat transfer between bodies, which are at rest, see Refs.<sup>7,8,20</sup>.

Non-equilibrium effects always prevail for bodies moving relative to each other. In Ref.<sup>6</sup> we used a dynamical modification of the Lifshitz theory to calculate the friction force between two parallel surfaces in relative motion (velocity  $V$ ). The calculation of the van der Waals friction is more complicated than of the Casimir-Lifshitz force (and of the radiative heat transfer), because it requires the determination of the electromagnetic field between moving boundaries. The solution can be found by writing the boundary conditions on the surface of each body in the rest reference frame of this body. The relation between the electromagnetic fields in the different reference frames is determined by the Lorentz transformation. In Ref.<sup>6</sup> the electromagnetic field in the vacuum gap between the bodies was calculated to linear order in  $V/c$ . These linear terms corresponds to mixing of electromagnetic waves with different polarizations. The waves with different polarization are statistically independent. Thus after averaging of the stress tensor over the fluctuating electromagnetic field, the mixing terms will give a contribution to the friction force of order  $(V/c)^2$ . In Ref.<sup>6</sup> the mixing terms were neglected, and the resulting formula for friction force is accurate to order  $(V/c)^2$ . The same approximation was used in Ref.<sup>21</sup> to calculate the frictional drag between quantum wells, and in Refs.<sup>22,23</sup> to calculate the friction force between plane parallel surfaces in normal relative motion. In Ref.<sup>24</sup> the correctness of the approach based on the dynamical modification of the Lifshitz theory was confirmed (at least to linear order in the sliding velocity  $V$ ) by rigorous quantum mechanical calculations (using the Kubo formula for friction coefficient). For recent reviews of the van der Waals friction see Refs.<sup>7,8</sup>.

In Ref.<sup>9</sup> we presented a of unified approach to the Casimir-Lifshitz forces and the radiative heat transfer at nonequilibrium conditions, when the bodies are at different temperatures, and move relative to each other with an arbitrary velocity  $V$ . In comparison with previous calculations<sup>6,21–23</sup>, we did not make any approximation in the Lorentz transformation of the electromagnetic field. Thus, we could determine the field in one inertial reference frame, knowing

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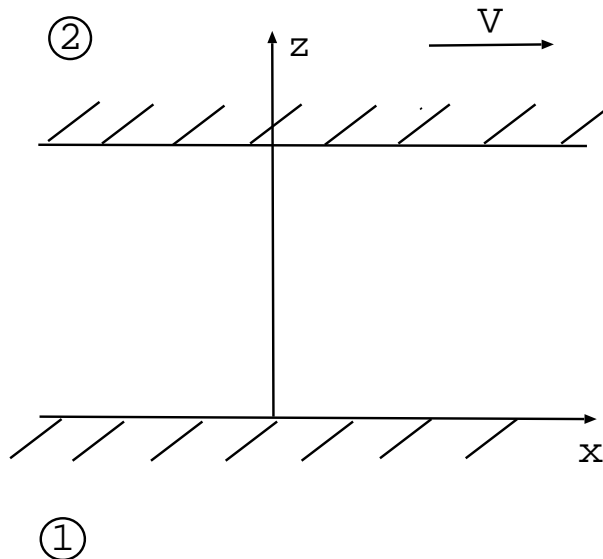


FIG. 1: Two semi-infinite bodies with plane parallel surfaces separated by a distance  $d$ . The upper solid moves parallel to the other solid with the velocity  $V$ .

the same field in another reference frame, and our solution of the electromagnetic problem was exact. Knowing the electromagnetic field we calculated the stress tensor and the Poynting vector which determined the Casimir-Lifshitz forces and the heat transfer, respectively. Upon going to the limit when one of the bodies is rarefied we obtained the interaction force and the heat transfer for a small particle-surface configuration.

In the recent papers Philbin and Leonhardt<sup>25,26</sup> (henceforth referred to as PL) calculated the Casimir-Lifshitz forces due to electromagnetic fluctuations between two perfectly flat parallel dielectric surfaces separated by vacuum and moving parallel to each other. In Ref.<sup>25</sup> LP used Lifshitz theory<sup>4,29,30</sup> and considered only the case of zero temperature. Lifshitz theory<sup>4,29,30</sup> also included the effect of thermal radiation in his analysis. The *Casimir-Lifshitz effect* is therefore also taken to describe forces that have a contribution from thermal radiation as well as from the quantum vacuum. The formalism developed by Lifshitz, however, cannot be used for plates at different temperatures. The general case of finite and different temperatures was considered in Ref.<sup>26</sup>. In Ref.<sup>26</sup> LP used the same approach as in Ref.<sup>9</sup>, which is based on a dynamical modification of the Rytov's theory. Theory from Ref.<sup>26</sup> contains as a limiting case the theory from Ref.<sup>25</sup>. For the contributions to the Casimir-Lifshitz forces resulting from thermal fluctuations LP obtained the same results as in Ref.<sup>9</sup>. However at zero temperature LP obtained a result which contradicts a substantial body of earlier results<sup>6-9,27</sup>. Their conclusion was that, at zero temperature, where only quantum fluctuations occur, friction is precisely zero. In this paper we argue for the correctness of the earlier results and point to the errors in the reasoning of PL.

## II. BASIC RESULTS

We consider two semi-infinite solids having flat parallel surfaces separated by a distance  $d$  and moving with velocity  $V$  relative to each other, see Fig. 1. We introduce two coordinate systems  $K$  and  $K'$  with coordinate axes  $xyz$  and  $x'y'z'$ . In the  $K$  system body **1** is at rest while body **2** moves with the velocity  $V$  along the  $x$ -axis. The  $xy$  and  $x'y'$  planes are in the surface of body **1**, and the  $x$  and  $x'$ -axes are parallel. The  $z$  and  $z'$ -axes point toward body **2**. In the  $K'$  system body **2** is at rest while body **1** is moving with velocity  $-V$  along the  $x$ -axis.

The force which acts on the surface of body **1** can be calculated from the Maxwell stress tensor  $\sigma_{ij}$ , evaluated at  $z = 0$ :

$$\sigma_{ij} = \frac{1}{4\pi} \int_0^\infty d\omega \int \frac{d^2q}{(2\pi)^2} \left[ \langle E_i E_j^* \rangle + \langle E_i^* E_j \rangle + \langle B_i B_j^* \rangle + \langle B_i^* B_j \rangle - \delta_{ij} (\langle \mathbf{E} \cdot \mathbf{E}^* \rangle + \langle \mathbf{B} \cdot \mathbf{B}^* \rangle) \right]_{z=0} \quad (1)$$

According to Ref.<sup>9</sup> the  $x$ -component of the force is given by

$$\begin{aligned}
F_x = \sigma_{xz} = & \frac{\hbar}{8\pi^3} \int_0^\infty d\omega \int_{q < \omega/c} d^2q \frac{q_x}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] \\
& \times [(q^2 - \beta k q_x)^2 (1 - |R_{1p}|^2)(1 - |R'_{2p}|^2) |D_{ss}|^2 \\
& + \beta^2 k_z^2 q_y^2 (1 - |R_{1p}|^2)(1 - |R'_{2s}|^2) |D_{sp}|^2 + (p \leftrightarrow s)] (n_2(\omega') - n_1(\omega)) \\
& + \frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q > \omega/c} d^2q \frac{q_x}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] e^{-2|k_z|d} \\
& \times [(q^2 - \beta k q_x)^2 \text{Im} R_{1p} \text{Im} R'_{2p} |D_{ss}|^2 - \beta^2 k_z^2 q_y^2 \text{Im} R_{1p} \text{Im} R'_{2s} |D_{sp}|^2 \\
& + (p \leftrightarrow s)] (n_2(\omega') - n_1(\omega)), \tag{2}
\end{aligned}$$

where  $R_{1p(s)} = R_{1p(s)}(\omega, q)$  and  $R'_{2p(s)} = R_{2p(s)}(\omega', q')$  are the reflection amplitudes for surfaces **1** and **2** for the  $p(s)$  - polarized electromagnetic field, respectively,  $\mathbf{q} = (q_x, q_y)$ ,  $k_z = ((\omega/c)^2 - q^2)^{1/2}$ ,  $\mathbf{q}' = (q'_x, q_y)$ ,  $q'_x = (q_x - \beta k)\gamma$ ,  $\omega' = (\omega - V q_x)\gamma$ ,  $\gamma = 1/\sqrt{1 - \beta^2}$ ,  $\beta = V/c$ ,  $k = \omega/c$ ,

$$q' = \gamma \sqrt{q^2 - 2\beta k q_x + \beta^2 (k^2 - q_y^2)},$$

$$D_{pp} = 1 - e^{2ik_z d} R_{1p} R'_{2p}, \quad D_{ss} = 1 - e^{2ik_z d} R_{1s} R'_{2s},$$

$$D_{sp} = 1 + e^{2ik_z d} R_{1s} R'_{2p}, \quad D_{ps} = 1 + e^{2ik_z d} R_{1p} R'_{2s},$$

$$\Delta = (q^2 - \beta k q_x)^2 D_{ss} D_{pp} + \beta^2 k_z^2 q_y^2 D_{ps} D_{sp},$$

$$n_i(\omega) = \frac{1}{e^{\hbar\omega/k_B T_i} - 1},$$

where  $T_1$  and  $T_2$  are the temperatures for bodies **1** and **2**, respectively. The symbol  $(p \leftrightarrow s)$  denotes the terms which can be obtained from the preceding terms by permutation of the indexes  $p$  and  $s$ . The first term in Eq. (2) represents the contribution to the friction from propagating waves ( $q < \omega/c$ ), and the second term from the evanescent waves ( $q > \omega/c$ ). If in Eq. (2) one neglects the terms of the order  $\beta^2$  then the contributions from waves with  $p$ - and  $s$ -polarization will be separated. In this case Eq. (2) is reduced to the formula obtained in Ref.<sup>6</sup>. Thus, to the order  $\beta^2$  the mixing of waves with different polarization can be neglected, what agrees with the results obtained in Ref.<sup>6</sup>. At  $T = 0$  K the propagating waves do not contribute to friction but the contribution from evanescent waves is not equal to zero. Taking into account that  $n(-\omega) = -1 - n(\omega)$  from Eq. (2) we get the friction mediated by the evanescent electromagnetic waves at zero temperature (in literature this type of friction is denoted as quantum friction<sup>27</sup>)

$$\begin{aligned}
F_x = & -\frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x \int_0^{q_x V} d\omega \frac{q_x}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] e^{-2|k_z|d} \\
& \times [(q^2 - \beta k q_x)^2 \text{Im} R_{1p} \text{Im} R'_{2p} |D_{ss}|^2 - \beta^2 k_z^2 q_y^2 \text{Im} R_{1p} \text{Im} R'_{2s} |D_{sp}|^2 + (p \leftrightarrow s)]. \tag{3}
\end{aligned}$$

In Ref.<sup>9</sup> some additional terms were overlooked in the equation for the  $z$ -component of the force. The correct form of this equation is given by<sup>28</sup>

$$F_z = \sigma_{zz} = -\frac{\hbar}{4\pi^3} \text{Re} \int_0^\infty d\omega \int d^2q \frac{k_z}{\Delta} e^{2ik_z d} \left\{ (q^2 - \beta k q_x)^2 [R_{1p} R'_{2p} D_{ss} \right.$$

$$\begin{aligned}
& + R_{1s} R'_{2s} D_{pp}] - \beta^2 k_z^2 q_y^2 [R_{1p} R'_{2s} D_{sp} + R_{1s} R'_{2p} D_{ps}] \Big\} [1 + n_1(\omega) + n_2(\omega')] \\
& - \frac{\hbar}{16\pi^3} \int_0^\infty d\omega \int_{q < \omega/c} d^2 q \frac{k_z}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] \\
& \times \{ (q^2 - \beta k q_x)^2 [(1 - |R_{1p}|^2)(1 + |R'_{2p}|^2) |D_{ss}|^2 - (1 \leftrightarrow 2, \omega \leftrightarrow \omega')] \\
& + \beta^2 k_z^2 q_y^2 [(1 - |R_{1p}|^2)(1 + |R'_{2s}|^2) |D_{sp}|^2 - (1 \leftrightarrow 2, \omega \leftrightarrow \omega')] + (p \leftrightarrow s) \} (n_1(\omega) - n_2(\omega')) \\
& + \frac{\hbar}{4\pi^3} \int_0^\infty d\omega \int_{q > \omega/c} d^2 q \frac{|k_z|}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] e^{-2|k_z|d} \\
& \times \{ (q^2 - \beta k q_x)^2 [\text{Im} R_{1p} \text{Re} R'_{2p} |D_{ss}|^2 - (1 \leftrightarrow 2, \omega \leftrightarrow \omega')] - \beta^2 k_z^2 q_y^2 [\text{Im} R_{1p} \text{Re} R'_{2s} |D_{sp}|^2 - (1 \leftrightarrow 2, \omega \leftrightarrow \omega')] \\
& + (p \leftrightarrow s) \} (n_1(\omega) - n_2(\omega')). \tag{4}
\end{aligned}$$

At  $T_1 = T_2 = 0$  K, Eq. (4) takes the form

$$\begin{aligned}
F_z = & -\frac{\hbar}{4\pi^3} \text{Re} \left\{ \int_0^\infty d\omega \int d^2 q - \int_{-\infty}^\infty dq_y \int_0^\infty dq_x \int_0^{q_x V} d\omega \right\} \frac{k_z}{\Delta} e^{2ik_z d} \\
& \left\{ (q^2 - \beta k q_x)^2 [R_{1p} R'_{2p} D_{ss} + R_{1s} R'_{2s} D_{pp}] \right. \\
& \left. - \beta^2 k_z^2 q_y^2 [R_{1p} R'_{2s} D_{sp} + R_{1s} R'_{2p} D_{ps}] \right\} \\
& + \frac{\hbar}{4\pi^3} \int_{-\infty}^\infty dq_y \int_0^\infty dq_x \int_0^{q_x V} d\omega \frac{|k_z|}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] e^{-2|k_z|d} \\
& \times \{ (q^2 - \beta k q_x)^2 [\text{Im} R_{1p} \text{Re} R'_{2p} |D_{ss}|^2 - (1 \leftrightarrow 2, \omega \leftrightarrow \omega')] - \beta^2 k_z^2 q_y^2 [\text{Im} R_{1p} \text{Re} R'_{2s} |D_{sp}|^2 - (1 \leftrightarrow 2, \omega \leftrightarrow \omega')] \\
& + (p \leftrightarrow s) \}. \tag{5}
\end{aligned}$$

The radiative energy transfer between the bodies is determined by the ensemble average of the Poynting's vector. In the case of two plane parallel surfaces  $z$ -component of the Poynting's vector is given by<sup>7</sup>

$$\begin{aligned}
\langle \mathbf{S}_{1z}(\mathbf{r}) \rangle_\omega & = (c/8\pi) \langle \mathbf{E}(\mathbf{r}) \times \mathbf{B}^*(\mathbf{r}) \rangle_\omega + c.c. \\
& = \frac{ic^2}{8\pi\omega} \left\{ \langle \mathbf{E}(\mathbf{r}) \cdot \frac{d}{dz} \mathbf{E}^*(\mathbf{r}) \rangle - c.c. \right\}_{z=0}. \tag{6}
\end{aligned}$$

According to Ref.<sup>9</sup> the heat flux across the surface **1** is given by:

$$\begin{aligned}
S_1 = & \frac{\hbar}{8\pi^3} \int_0^\infty d\omega \int_{q < \omega/c} d^2 q \frac{\omega}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] \\
& \times [(q^2 - \beta k q_x)^2 (1 - |R_{1p}|^2)(1 - |R'_{2p}|^2) |D_{ss}|^2 \\
& + \beta^2 k_z^2 q_y^2 (1 - |R_{1p}|^2)(1 - |R'_{2s}|^2) |D_{sp}|^2 + (p \leftrightarrow s)] (n_2(\omega') - n_1(\omega)) \\
& + \frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q > \omega/c} d^2 q \frac{\omega}{|\Delta|^2} [(q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2] e^{-2|k_z|d} \\
& \times [(q^2 - \beta k q_x)^2 \text{Im} R_{1p} \text{Im} R'_{2p} |D_{ss}|^2 - \beta^2 k_z^2 q_y^2 \text{Im} R_{1p} \text{Im} R'_{2s} |D_{sp}|^2 \\
& + (p \leftrightarrow s)] (n_2(\omega') - n_1(\omega)). \tag{7}
\end{aligned}$$

### III. DISCUSSION AND COMPARISON WITH THE RESULTS OF PL

In Ref.<sup>25</sup> PL used Lifshitz theory and considered only the case of zero temperature. There are two variants of Lifshitz theory. In the first variant the Maxwell stress tensor is calculated using electromagnetic field which was calculated using Rytov's theory. In the second variant the Maxwell stress tensor is calculated using Green's functions of the electromagnetic field which were calculated from Maxwell's equations. Both these variants give the same results for forces. In Ref.<sup>25</sup> PL used the second variant of the Lifshitz theory. The general case of finite and different temperatures was considered by PL in Ref.<sup>26</sup>. The theory from Ref.<sup>26</sup> contains as limiting case the theory from Ref.<sup>25</sup>. In particular, in both these theories the authors came to conclusion that there is no lateral force on the plates in relative motion. In Ref.<sup>26</sup> PL used the same approach as by Volokitin *et al*<sup>9,28</sup> (henceforth referred to as VP) but they came to the opposite conclusion that there is no "quantum" friction. Between these two studies there is a difference only in the technical details. VP calculated total electromagnetic field in the rest reference frame of surface **1**. The total electromagnetic field contains the contributions from quantum and thermal fluctuations in both bodies. This electromagnetic field was used in the calculations of the stress tensor and the Poynting's vector in the rest reference frame of surface **1**. LP divided the total stress tensor into two contributions from surfaces **1** and **2**. The contribution from surface **2** was first calculated in the rest reference frame of surface **2**. The contribution from the surface **2** again was divided into two components - one from quantum and other from thermal fluctuations. Separation of quantum-vacuum from thermal effect is achieved by the identity

$$\coth\left(\frac{\hbar\omega}{2k_B T}\right) = \text{sgn}(\omega) + 2\text{sgn}(\omega) \left[ \exp\left(\frac{\hbar|\omega|}{k_B T_2}\right) - 1 \right]^{-1} \quad (8)$$

where the first term gives the quantum-vacuum part and the the second term, containing the Plank spectrum, gives the thermal radiation part. The Lorentz transformation for the stress tensor was used to obtain the contribution from thermal fluctuations in body **2** in the rest reference frame of surface **1**. The integrand of this contribution contains factor

$$2\text{sgn}(\omega') \left[ \exp\left(\frac{\hbar|\omega'|}{k_B T_2}\right) - 1 \right]^{-1} \quad (9)$$

As a result, for total contribution from thermal fluctuations in both bodies, PL obtained exactly the same results as it was obtained by VP. However, for the contribution from quantum fluctuations PL proposed that the Lorentz transformation is not valid, and arrived at the conclusion that the effect of zero-point radiation for contribution from plate **2** can be obtained by the following replacement of a factor in the integrand in the expression for contribution from thermal fluctuations:

$$2\text{sgn}(\omega') \left[ \exp\left(\frac{\hbar|\omega'|}{k_B T_2}\right) - 1 \right]^{-1} \rightarrow \text{sgn}(\omega) + 2\text{sgn}(\omega') \left[ \exp\left(\frac{\hbar|\omega'|}{k_B T_2}\right) - 1 \right]^{-1} \quad (10)$$

Contribution from plate **1** is given by a similar expression as from plate **2**. As a result, for friction force PL obtained expression which is similar to Eq. (2) but with replacement

$$(n_2(\omega') - n_1(\omega)) \rightarrow (\text{sgn}(\omega')n_2(|\omega'|) - n_1(\omega)). \quad (11)$$

For finite temperatures PL obtained the same contribution to friction from thermal fluctuations as by VP. However, it is clear from Eq. (11) that at  $T = 0$  K the factor on the right side of Eq. (11) is equal to zero what leads PL to the conclusion that there is no lateral force at zero temperature. PL claimed that the vanishing of the lateral force at zero temperature can be viewed as a consequence of the Lorentz invariance of the quantum zero-point radiation—it has the same "spectrum" in every inertial reference frame. However, instead of proving this PL just postulated the existence of such an invariance. If the Lorentz transformation is used also to calculate contribution to stress tensor from quantum fluctuations in plate **2**, in the rest reference frame of plate **1**, then instead of the factor given by the right side of Eq. (10), in the integrand will occur the factor

$$\text{sgn}(\omega') + 2\text{sgn}(\omega') \left[ \exp\left(\frac{\hbar|\omega'|}{k_B T_2}\right) - 1 \right]^{-1} = 1 + 2n_2(\omega'), \quad (12)$$

which will result in the friction force given by Eq. (2). For propagating waves  $\text{sgn}(\omega') = \text{sgn}(\omega)$  (for  $\omega > 0$ ) and from Eq. (2) it follows that the contribution to friction from propagating waves is equal to zero at zero temperature, which agrees with the principle of relativity. However, the contribution from evanescent waves is not equal to zero even at

zero temperature because in this case  $\text{sgn}(\omega') < 0$  for  $\omega < q_x V$ . PL claim that maybe the Lorentz transformation for the stress tensor is not valid for the contribution to stress tensor from quantum fluctuations. However, VP, instead of using Lorentz transformation for the stress tensor, apply this transformation for calculation of the electromagnetic field. This electromagnetic field is used to calculate the stress tensor in the rest reference frame of plate 1. The result was the same. Thus contrary to the opinion of PL, we argue that the Lorentz invariance exist only for quantum fluctuations corresponding to propagating waves. This means that the spectral characteristics of the electromagnetic field in absolute vacuum (without of any bodies) are the same in all inertial reference frames; otherwise it will contradict to the principle of relativity. This result follows from the Lorentz transformation for the electromagnetic field corresponding to the propagating electromagnetic waves. For evanescent waves there is no Lorentz invariance of the spectral properties of the electromagnetic field. This result, which also follows from the Lorentz transformation, does not contradict to the principle of relativity because there are no evanescent waves in absolute vacuum.

At zero temperature the integration in Eq. (2) includes only the interval  $0 < \omega < q_x V$ . This integration takes into account the contribution to friction from excitations in this frequency range, which exist even at zero temperature. PL did not include these excitations, and as a result they got zero friction. Excitations which exist even at zero temperature contribute not only to the lateral force, but also to normal force (see Eq. (4)). Thus the conservative Casimir-Lifshitz force also contains some additional terms which were overlooked by PL.

Recently Pendry<sup>31</sup> has also showed that the friction is finite even at zero temperature, in qualitative agreement with most previous approaches to the problem, but in contradiction to the conclusion of PL. However Pendry considered a very simple non-retarded and non-relativistic model. In contrast to Pendry, in the framework of the same model we show that the calculation of PL is in error. We show that this error is due to the assumption that the zero-point radiation, corresponding to evanescent electromagnetic waves, obey Lorentz invariance. We also show that normal component of Casimir-Lifshitz force calculated by PL is also incorrect.

Since theory from Ref.<sup>25</sup> is a limiting case of theory from Ref.<sup>26</sup>, our Comment is applicable for both these papers.

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